Worksheet 2 solution

(1) $\frac{0^2+8.0+1}{4.0^2-5} = \frac{1}{5}$

Now, divide by x^2 each term, $\lim_{x\to+\infty} \frac{1+8/x+1/x^2}{4-5/x^2} = \frac{1}{4}$ since $\lim_{x\to+\infty} \frac{1}{x} = 0$

(2) since $\frac{3x^3-2x}{x^3-\pi x^2}$ at 0 is $\frac{0}{0}$, we can either do L'Hôpital or

$= \frac{3x^2-2}{x^2-\pi x}$. Then the limit is $\frac{2}{0}$ => Undefined

Similarly on the second limit, by L'Hôpital $= \lim_{x\to 3} \frac{2x-6}{3x^2-9} = 0$

(3) $f'(4) = \lim_{a\to 4} \frac{a^2-4^2}{a-4} = \lim_{a\to 4} (a+4) = 8$

$\Rightarrow f'(4).g'(1) = 24$

$g'(1) = \lim_{a\to 1} \frac{a^3-1^3}{a-1} = \lim_{a\to 1} a^2+a+1 = 3$

(4) If $a_0 = 0 \Rightarrow a_n = 0$, then the limit is 0
If $a_0 = 1 \Rightarrow a_n = 1$, then the limit is 1
If $a_0 > 1$, the sequence grows to $+\infty$ => the limit is undefined
If $a_0 < 1$, the sequence decreases to 0 => the limit is 0

(5) Since $H(x) = 1$ for $x > 0$, $\lim_{x\to 0^+} H(x) = 1$
Since $H(x) = 0$ for $x < 0$, $\lim_{x\to 0^-} H(x) = 0$